



# RESEARCH MEMORANDUM

ANALYSIS OF FULLY DEVELOPED TURBULENT HEAT TRANSFER AT LOW  
PECLET NUMBERS IN SMOOTH TUBES WITH APPLICATION TO LIQUID  
METALS

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NATIONAL ADVISORY COMMITTEE  
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RESEARCH MEMORANDUMANALYSIS OF FULLY DEVELOPED TURBULENT HEAT TRANSFER AT LOW PECKET NUMBERS  
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## SUMMARY

An analysis was made of heat transfer for fluids flowing turbulently at low Peclet numbers in smooth tubes. Previous analyses for flow of gases and liquid metals at low Peclet numbers gave higher heat-transfer coefficients than were indicated experimentally. When the mixing-length theory was modified in order to account for the heat transferred by conduction to a turbulent particle as it moves transversely, the predicted results were brought into agreement with the experimental results.

## INTRODUCTION

Analyses of heat transfer for liquid metals flowing turbulently in smooth tubes (low Prandtl numbers) are given in references 1 and 2. The predicted heat-transfer coefficients from these analyses are considerably higher than those determined in the experimental heat-transfer investigations for mercury and lead bismuth given in references 2 to 4. It should be noted, however, that the experimental data were all obtained at low Peclet numbers. If the analytical and experimental results for flow of gases are compared at low Peclet numbers, they are also found to be in disagreement (references 5 and 6). These results indicate that the eddy diffusivities for momentum and heat transfer cannot be considered equal at low values of Peclet number. The fact that the eddy diffusivities for heat and momentum transfer cannot be considered equal at low Peclet numbers is also illustrated by the temperature distributions given in reference 5 where a considerable separation of data was noted at low Peclet numbers. Analytical and experimental investigations of the variation of the ratio of eddy diffusivities for heat and momentum transfer are given in references 7 and 8. In an analysis made at the NACA Lewis laboratory, the heat transferred by conduction to a turbulent particle as it moves transversely is considered so that the eddy diffusivity for heat transfer is reduced to a value below that for momentum transfer. Although a simplified theory is used in the analysis, it appears to be sufficiently accurate for predicting heat transfer at low Peclet numbers.

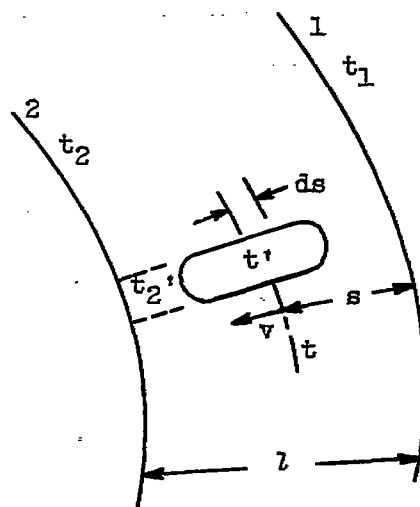
## ANALYSIS

When turbulent flow exists in a tube, portions of the fluid move about in random fashion. If heat transfer between the tube wall and the fluid takes place, a temperature gradient occurs across the tube and some of the portions of the fluid move transversely into regions of different mean temperature. This motion produces heat transfer in addition to the transfer that takes place by local molecular conduction.

In the usual mixing-length theory of turbulent heat transfer, an eddy or turbulent particle is assumed to originate from an instability at a point in the fluid having the mean temperature of the fluid at the point, to travel transversely at constant temperature a distance of one mixing length, and then to mix with the fluid and assume the average temperature of the fluid at the point of mixing (reference 9). (In the present discussion, an eddy or turbulent particle is considered to be a portion of fluid all parts of which move with approximately the same velocity.) For fluids having very high thermal conductivities, such as liquid metals, however, heat is probably conducted to or from the particle as it moves along its path. This should also occur for fluids having low conductivities if the turbulence is low. In general, this effect is to be expected if the ratio of eddy diffusivity to thermal diffusivity is small. Because the Peclet number  $u_p D / (k / \rho g c_p)$  is a measure of the ratio of the average eddy diffusivity at a cross section of the tube to the thermal diffusivity, this effect should occur at low Peclet numbers. (All symbols are defined in the appendix.) In the following analysis an attempt is made to account for the heat conducted to or from a turbulent particle as it moves along its path. An analysis using somewhat the same general approach is given in reference 7, but the assumptions and results differ from those of the present analysis. The assumptions used in reference 7 required the eddy diffusivities for heat and momentum transfer to be equal at a Prandtl number of 1, whereas experimental evidence indicates that this is not always the case. In the present analysis, the eddy diffusivity for heat transfer is lower than the eddy diffusivity for momentum transfer at low Peclet numbers for a Prandtl number of 1.

## Modified Mixing-Length Theory of Heat Transfer

In the following analysis, eddies are assumed to move transversely in the tube between concentric cylindrical surfaces 1 and 2 which are separated by the small distance  $\ell$ , the average mixing length at a particular radius in the tube. Eddies also move longitudinally but these



should not affect the radial heat transfer. The eddies originate from an instability at either surface 1 or 2 and travel to the other surface where they mix with the fluid. The mixing might be caused by collisions between eddies.

Equation for turbulent heat transfer. - The case of heat transfer from the tube wall to the fluid is considered herein. An eddy of fluid originates at the outer surface 1 where it has the time average temperature of the fluid at surface 1  $t_1$  and arrives at surface 2 with the lower temperature  $t_2'$  where it mixes with the fluid and assumes the time average temperature of the fluid at surface 2  $t_2$ . In general,  $t_2'$  differs from  $t_1$  because of heat conducted out of the eddy between surfaces 1 and 2. The heat transported from surface 1 to surface 2 by the eddy is equal to the mass of the eddy multiplied by  $c_p g(t_2' - t_2)$ . The turbulent heat transfer per unit area per unit time from surface 1 to surface 2 is then given by

$$q_t = \rho g c_p f v (t_2' - t_2) \quad (1)$$

where  $f$  is the fraction of surface 2 on which the fluid in the eddies makes contact and  $v$  is the average velocity of this fluid toward surface 2. Equation (1) can be written as

$$q_t = \rho g c_p f v \alpha (t_1 - t_2) \quad (2)$$

where

$$\alpha = \frac{t_2' - t_2}{t_1 - t_2} \quad (3)$$

Inasmuch as  $l$  is assumed to be small, the temperature gradient  $dt/dy$  is approximately constant over  $l$  and  $t_1 - t_2$  can be replaced by  $-l dt/dy$ . Equation (2) then becomes

$$q_t = -\rho g c_p (fvl) \propto \frac{dt}{dy} \quad (4)$$

or

$$q_t = -\rho g c_p \epsilon \frac{dt}{dy} \quad (5)$$

where  $\epsilon$  is the coefficient of eddy diffusivity, the value for which depends on the amount and kind of turbulent mixing at a point ( $\epsilon \alpha$  is sometimes replaced by  $\epsilon_h$ , the eddy diffusivity for heat transfer).

Equation for turbulent shear stress. - With the use of the usual assumption of the mixing-length theory, that is, the component of the velocity of an eddy in the direction along the axis of the tube does not change as the eddy moves between surfaces 1 and 2, the following equation for the turbulent shear stress is obtained:

$$\tau_t = \rho (fvl) \frac{du}{dy} \quad (6)$$

or

$$\tau_t = \rho \epsilon \frac{du}{dy} \quad (7)$$

where the eddy diffusivities  $\epsilon$  in equations (5) and (7) are identical. The assumption that they can be considered equal for fully developed flow in tubes is further supported by the agreement between the analysis and the experiment in the range of Peclet numbers where  $\alpha$  equals 1 (reference 5):

Determination of  $\alpha$ . - In order to obtain a relation for  $\alpha$  in equation (5) as defined by equation (3), the value for  $t_2'$  must be determined. Let  $t'$  be the temperature of a point on the eddy at some distance  $s$  from surface 1.

The heat transfer per unit area from the eddy at position  $s$  is equal to  $h'(t' - t)$  where  $h'$  is the coefficient of heat transfer between the eddy and the surrounding fluid. Inasmuch as the dimensions and velocity of the eddy are small, the Reynolds number is small and laminar heat transfer is assumed. Therefore,  $h'$  is proportional to  $k/\delta$ , where  $\delta$  is the film thickness or boundary layer associated with the heat transfer from the eddy. Then the heat transferred from an eddy at  $s$  in length  $ds$  is  $kp ds(t' - t)/\delta$  where  $p$  is the perimeter

of the eddy. This heat transfer decreases the temperature of the eddy in the direction  $s$  and can also be written as  $-\rho g c_p v A' (dt'/ds) ds$ , where  $A'$  is the cross-sectional area of the eddy perpendicular to the direction  $s$ . Equating these two expressions for heat transfer gives

$$\frac{dt'}{ds} = - \frac{kp}{\rho g c_p v A' \delta} (t' - t) \quad (8)$$

As before,  $dt/dy$  is considered constant over  $l$  so that

$$t = t_1 + s \frac{dt}{dy} \quad (9)$$

Substitution of equation (9) in equation (8) and rearrangement gives

$$\frac{dt'}{ds} + \left( \frac{kp}{\rho g c_p v A' \delta} \right) t' = \left( \frac{kp}{\rho g c_p v A' \delta} \right) \left( t_1 + \frac{dt}{dy} s \right) \quad (10)$$

Equation (10) can be written as

$$\frac{dt'}{ds} + \frac{p}{A' Pe'} t' = \frac{p}{A' Pe'} \left( t_1 + \frac{dt}{dy} s \right) \quad (11)$$

where  $Pe'$  is the Peclet number for the eddy. Equation (11) is a first order linear differential equation in the variables  $t'$  and  $s$ . The solution of equation (11) with the initial condition that  $t' = t_1$  when  $s = 0$  is

$$t' = s \frac{dt}{dy} - \frac{A' Pe'}{p} \frac{dt}{dy} + t_1 + \frac{A' Pe'}{p} \frac{dt}{dy} e^{-\left( \frac{p}{A' Pe'} \right) s} \quad (12)$$

For  $s = l$ ,  $t' = t_2'$ , and also  $t_1 - t_2 = -l \frac{dt}{dy}$ , then

$$\frac{t_2' - t_2}{t_1 - t_2} = \alpha = \frac{A' Pe'}{lp} \left[ 1 - e^{-\left( \frac{lp}{A' Pe'} \right)} \right] \quad (13)$$

In order to use equation (13), it is necessary to make some assumptions concerning the quantities  $p$ ,  $A'$ , and  $Pe'$ . The first assumption made is that the dimensions of the eddy are proportional to

the mixing length  $l$ , or  $A'$  is proportional to  $p^2$  which is proportional to  $l^2$ . An assumption must also be made for  $Pe'$ . As a first approximation,  $Pe'$  is assumed proportional to  $Pe$ , the Peclet number for the tube, and the results will be compared with experimental results. Equation (13) becomes, by introduction of these assumptions,

$$\alpha = b Pe \left[ 1 - e^{-\left(\frac{1}{b Pe}\right)} \right] \quad (14)$$

where  $b$  is a constant to be determined experimentally. Equation (14) gives the desired relation for  $\alpha$  to be substituted in equation (5).

#### Calculation of Nusselt Number, Reynolds Numbers, and Peclet Number for Uniform Heat Transfer and Uniform Fluid Properties

When the molecular shear stress is added to the turbulent shear stress given by equation (7) and the molecular heat transfer is added to the turbulent heat transfer given by equation (5), the following equations for shear stress and heat transfer per unit area are obtained:

$$\tau = (\mu + \rho \epsilon) \frac{du}{dy} \quad (15)$$

$$q = - (k + \rho g c_p \epsilon \alpha) \frac{dt}{dy} \quad (16)$$

where  $\alpha$  is given by equation (14). The eddy diffusivity  $\epsilon$  is obtained from the following expressions which are experimentally verified in references 5 and 10:

$$\epsilon = n^2 u y \quad (17)$$

for flow close to the wall ( $y^+ < 26$ ) and the Kármán relation

$$\epsilon = \kappa^2 \frac{(du/dy)^3}{(d^2u/dy^2)} \quad (18)$$

for flow at a distance from the wall ( $y^+ > 26$ ), where  $n$  and  $\kappa$  are experimental constants having the values 0.109 and 0.36, respectively.

By integration of equation (15) for the region at a distance from the wall with the viscous shear stress neglected, von Kármán obtained, for

uniform properties, the following dimensionless equation (reference 10):

$$u^+ = \frac{1}{\kappa} \left[ \sqrt{1 - \frac{y^+}{r_0^+}} + \log_e \left( 1 - \sqrt{1 - \frac{y^+}{r_0^+}} \right) \right] + C \quad (19)$$

Equation (18) becomes, on substitution of the first and second derivatives from equation (19),

$$\frac{\epsilon}{\mu/\rho} = 2 \kappa r_0^+ \left( 1 - \frac{y^+}{r_0^+} \right) \left( 1 - \sqrt{1 - \frac{y^+}{r_0^+}} \right) \quad (20)$$

for flow at a distance from the wall. For flow close to the wall, equation (17) can be written in dimensionless form as

$$\frac{\epsilon}{\mu/\rho} = n^2 u^+ y^+ \quad (21)$$

where the values of  $u^+$  are obtained from the generalized velocity distributions (constant fluid properties) given in reference 10. Values of  $\epsilon$  from equations (20) and (21) are substituted into equation (16) for obtaining temperature distributions. Equation (16) can be written in dimensionless form as

$$\frac{q}{q_0} = - \left[ 1 + \frac{Pe}{2} \alpha \epsilon' \right] \frac{dt''}{dy'} \quad (22)$$

The dimensionless quantities are defined in the list of symbols. Equation (22) can be written in integral form for calculating temperature distributions as

$$t_0'' - t'' = \int_0^{y'} \frac{q/q_0}{1 + \frac{Pe}{2} \alpha \epsilon'} dy' \quad (23)$$

where

$$\epsilon' \equiv \frac{\epsilon}{u_b r_0} = 2 \frac{\epsilon}{(\mu/\rho) Re} \quad (24)$$

The expression for  $Re$  will be given in the section Reynolds and Peclet numbers. An equation for  $q/q_0$  will be obtained in the following section.



Variation of  $q/q_0$  across tube. - The variation of heat transfer per unit area across the tube can be obtained by taking a heat balance on an annulus of fluid having differential length  $dx$ , outside radius  $r$ , and thickness  $dr$ . The heat added to the annulus must be equal to the heat transferred into the annulus minus the heat transferred out of the annulus, or

$$2\pi r dr \rho g u_{cp} \frac{dt}{dx} = 2\pi r q - 2\pi(r - dr)(q - dq)$$

or, neglecting higher order differentials,

$$d(rq) = \rho g u_{cp} \frac{dt}{dx} r dr \quad (25)$$

If  $dt/dx$  is constant across the tube, as must be the case for a fully developed temperature distribution with uniform heat transfer, the following heat balance can be written for the cross section as a whole:

$$\pi r_0^2 \rho g u_{bc} \frac{dt}{dx} = 2\pi r_0 q_0 \quad (26)$$

Dividing equation (25) by equation (26) gives

$$\frac{u}{u_b} \frac{r}{r_0} dr = \frac{d(rq)}{2q_0} \quad (27)$$

Replacing  $r$  by  $(r_0 - y)$  and writing equation (27) in dimensionless form yields

$$d \left[ (r_0^+ - y^+) \frac{q}{q_0} \right] = - \frac{4u^+}{Re} \frac{(r_0^+ - y^+)}{Re} dy^+ \quad (28)$$

Integrating gives

$$\int_{r_0^+}^{(r_0^+ - y^+) q/q_0} d \left[ (r_0^+ - y^+) \frac{q}{q_0} \right] = - \frac{4}{Re} \int_0^{y^+} u^+(r_0^+ - y^+) dy^+$$

or

$$\frac{q}{q_0} = \frac{r_0^+}{r_0^+ - y^+} - \frac{4}{Re(r_0^+ - y^+)} \int_0^{y^+} u^+(r_0^+ - y^+) dy^+ \quad (29)$$

Equation (29) gives the desired relation which is to be substituted into equation (23) for the variation of  $q/q_0$  across the tube.

Nusselt numbers. - The bulk temperature  $t_b$  for constant fluid properties is defined by

$$t_b = \frac{\int_0^{r_0} t u r \, dr}{\int_0^{r_0} u r \, dr} \quad (30)$$

Equation (30) can be written in dimensionless form as

$$t_0'' - t_b'' = \frac{\int_0^1 (t_0'' - t'') u^+(1 - y') dy'}{\int_0^1 u^+(1 - y') dy'} \quad (31)$$

where  $t_0'' - t''$  is obtained from equation (23), but

$$h \equiv \frac{q_0}{t_0 - t_b} \quad (32)$$

therefore

$$Nu \equiv \frac{hD}{k} = \frac{2}{t_0'' - t_b''} \quad (33)$$

Reynolds and Peclet numbers. - The Reynolds number  $Re$  is given by

$$Re \equiv \frac{\rho u_b D}{\mu} = 2u_b^+ r_0^+ \quad (34)$$

where  $u_b^+$  is given in reference 5 as

$$u_b^+ = \frac{2}{(r_0^+)^2} \int_0^{r_0^+} u^+(r_0^+ - y^+) dy^+ \quad (35)$$

The Peclet number is calculated from

$$Pe = (Re)(Pr) \quad (36)$$

Calculation of Nusselt number as a function of Reynolds or Peclet number and Prandtl number. - In the calculation of Nusselt number, the values for  $r_0^+$  and  $Pr$  are to be assumed first. With the use of the  $u^+$  against  $y^+$  curve from reference 10 (fig. 1),  $u_b^+$  can be calculated from equation (35). Then  $Re$  can be calculated from equation (34) and  $Pe$  from equation (36). The next step is to obtain  $\epsilon/(\mu/\rho)$  as a function of  $y^+$  from equations (20) and (21), and  $\epsilon'$  from equation (24). Then,  $q/q_0$  is obtained from equation (29) and  $y'$  from  $y' = y^+/r_0^+$  for each value of  $y^+$ . The quantity  $\alpha$  can be calculated from equation (14). The generalized temperature difference  $t_0'' - t''$  can then be calculated as a function of  $y'$  from equation (23). With the temperature distribution known,  $t_0'' - t_b''$  can be obtained from equation (31) and the Nusselt number from equation (33).

## RESULTS AND DISCUSSION

Variation of heat transfer per unit area across tube for uniform heat transfer at surface. - The variation of heat transfer per unit area with  $y^+$  for various values of  $r_0^+$  is shown in figure 2. The curves indicate that the heat transfer per unit area varies nonlinearly from a maximum at a point near the wall to zero at the tube center. For low values of  $r_0^+$ , the maximum heat-transfer per unit area is attained at a point slightly removed from the wall rather than at the wall. This is because the heat-transfer area decreases as the distance from the wall increases and thus, even though the total heat transfer decreases as the distance from the wall increases, the heat transfer per unit area increases slightly for some conditions.

Experimental determination of constant  $b$ ; Nusselt numbers for gases. - In order to calculate relations among Nusselt number, Reynolds or Peclet number, and Prandtl number; it is necessary to determine experimentally the constant  $b$  in equation (14). This constant is obtained from experimental data for heat transfer to air in reference 5 and plotted in figure 3. Inasmuch as the data were obtained at high ratios of wall to fluid bulk temperature, the temperature at which the fluid properties in the Nusselt and Reynolds numbers are evaluated is important. The data indicate that, for Reynolds numbers below 15,000, the best correlation was obtained by evaluating the properties at the bulk temperature; for Reynolds numbers between 15,000 and 30,000, equally good correlation was

was obtained by evaluating the properties at the bulk temperature or at  $t_{0.4}$  where  $t_{0.4} = 0.4(t_0 - t_p) + t_p$ ; and for Reynolds numbers above 30,000, the best correlation was obtained by evaluating the properties at  $t_{0.4}$ . In figure 3 the properties are evaluated at the bulk temperature for Reynolds numbers below 30,000 and at  $t_{0.4}$  for Reynolds numbers above 30,000. A justification for evaluating the properties at the bulk temperature rather than at  $t_{0.4}$  for the lower Reynolds numbers is found in reference 11, in which it is shown that for laminar flow the properties should be evaluated at a temperature between the bulk temperature and the temperature at the center of the tube. It would then be expected that in the transition region the properties should be evaluated at a temperature between this temperature and  $t_{0.4}$ .

Predicted curves from the analysis are included in figure 3. It is apparent that for  $\alpha = 1$  (dashed curve), the predicted Nusselt numbers at low Reynolds or Peclet numbers are too high, whereas the solid curve for which  $\alpha$  is calculated from equation (14) is in good agreement with the experimental results. The value of the constant  $b$  in equation (14) is 0.000153 as determined from the experimental data.

In order for the analysis to apply, the Reynolds number should be well above the Reynolds numbers at which transition from laminar to turbulent flow takes place. The velocity-distribution data for air (fig. 1) indicate no effect of transition for Reynolds numbers as low as 8000. On the other hand, the temperature-distribution data for air from reference 5 indicate a considerable effect at that Reynolds number so that the data in figure 3 at Reynolds numbers in the vicinity of 10,000 should be suitable for determining the constant  $\alpha$ .

Liquid-metal heat transfer. - Predicted curves for Nusselt number against Peclet number for a Prandtl number of 0.01 are shown in figure 4. The curve for  $\alpha = 1$  is substantially the same as those obtained in references 1 and 2. The solid curve was obtained by computing  $\alpha$  from equation (14) where the value of  $b$  (0.000153) was obtained from the data for flow of air in figure 2. Curves representing data for mercury (data obtained from Musser and Page, reference 2) and lead bismuth from reference 3, are also shown. It is shown in reference 2 that for Prandtl numbers below 0.1 the curve of Nusselt number against Peclet number is practically independent of Prandtl number so that the predicted curves for  $Pr = 0.01$  should be comparable to the data for mercury and lead bismuth. It is apparent from figure 4 there is improvement in the agreement between experimentally and analytically determined values when  $\alpha$  is computed from equation (14) although there is considerable scatter in the data. Data at higher Peclet numbers, in the region where the two predicted curves begin to merge, would be of value in order to establish further the validity of the analysis. This data would indicate whether or not  $\alpha$  actually approaches 1 at high Peclet number for liquid metals.

An empirical equation of the type given in reference 2 is also plotted in figure 4. This equation is

$$Nu = 6.3 + 0.000222 (Pe)^{1.3} \quad (37)$$

Equation (37) represents the predicted line with  $\alpha$  computed from equation (14) with good accuracy for the range of Peclet numbers shown in figure 4.

#### SUMMARY OF RESULTS

The following results were obtained from the analytical investigation of turbulent heat transfer at low Peclet numbers:

1. The mixing-length theory was modified in order to include the heat conducted to or from an eddy as it moves transversely. In order to utilize the analysis, a constant was determined experimentally.
2. When the experimental constant was determined from a point on the curve of Nusselt number for gases, the analytical results were brought into better agreement with the data for both gases and liquid metals at low Peclet numbers.

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## APPENDIX - SYMBOLS

A'	cross-sectional area of eddy perpendicular to radial direction, $\text{ft}^2$
b	experimental constant
C	constant of integration
$c_p$	specific heat at constant pressure, $\text{Btu/lb } (^\circ\text{F})$
D	inside diameter of tube, $\text{ft}$
f	fraction of surface 1 or 2 on which eddies are arriving from the other surface
g	acceleration due to gravity, $32.2 \text{ ft/sec}^2$
h	coefficient of heat transfer between wall and fluid, $q_0/(t_0 - t_b)$ , $\text{Btu}/(\text{sec})(\text{ft}^2)(^\circ\text{F})$
h'	coefficient of heat transfer between eddy and fluid, $\text{Btu}/(\text{sec})(\text{ft}^2)(^\circ\text{F})$
k	thermal conductivity of fluid, $(\text{Btu})(\text{ft})/(\text{sec})(\text{ft}^2)(^\circ\text{F})$
l	average mixing length at a particular radius, $\text{ft}$
Nu	Nusselt number, $hD/k$
n	experimental constant
Pe	Peclet number for tube, $u_b D / [k/(\rho g c_p)]$
Pe'	Peclet number for eddy, $v \delta / [k/(\rho g c_p)]$
Pr	Prandtl number, $c_p g \mu / k$
p	perimeter of eddy, $\text{ft}$
q	total rate of heat transfer per unit area toward tube center, $(\text{Btu}/(\text{sec})(\text{ft}^2))$
$q_t$	rate of heat transfer per unit area toward tube center by turbulent motion, $\text{Btu}/(\text{sec})(\text{ft}^2)$
$q_0$	total rate of heat transfer at wall toward tube center per unit area, $\text{Btu}/(\text{sec})(\text{ft}^2)$
Re	Reynolds number, $\rho u_b D / \mu$

$r$	radius, distance from tube axis, ft
$r_0$	inside tube radius, ft
$r_0^+$	tube radius parameter, $\frac{\sqrt{\tau_0/\rho}}{\mu/\rho} r_0$
$s$	distance toward tube center from surface 1 where eddy originates, ft
$t$	time average temperature at a point, °F
$t_0$	wall temperature, °F
$t_{0.4}$	$0.4 (t_0 - t_b) + t_b$
$t_1$	time average temperature at surface 1, where eddy originates, °F
$t_2$	time average temperature at surface 2, where eddy mixes with fluid, °F
$t'$	temperature of a point on eddy a distance $s$ from surface 1, °F
$t_2'$	temperature of eddy at surface 2, °F
$t_b$	bulk or average temperature at a cross section of tube, °F
$t''$	dimensionless temperature parameter, $(k/q_0 r_0)t$
$t_0''$	dimensionless wall temperature parameter, $(k/q_0 r_0)t_0$
$t_b''$	dimensionless bulk temperature parameter, $(k/q_0 r_0)t_b$
$u$	time average axial velocity at a point in fluid, ft/sec
$u_b$	bulk or average velocity at a cross section in tube, ft/sec
$u^+$	velocity parameter, $u/\sqrt{\tau_0/\rho}$
$u_b^+$	bulk velocity parameter, $u_b/\sqrt{\tau_0/\rho}$
$v$	radial velocity of eddy, ft/sec
$x$	distance from tube entrance, ft
$y$	distance from tube wall, ft
$y^+$	wall distance parameter, $\frac{\sqrt{\tau_0/\rho}}{\mu/\rho} y$

$y'$	$y/r_0$
$\alpha$	$(t_2' - t_2)/(t_1 - t_2), \epsilon/\epsilon_h$
$\delta$	film thickness or thickness of boundary layer on eddy, ft
$\epsilon$	coefficient of eddy diffusivity for momentum, $\text{ft}^2/\text{sec}$
$\epsilon_h$	coefficient of eddy diffusivity for heat transfer, $\text{ft}^2/\text{sec}$
$\epsilon'$	$\epsilon/(u_b r_0)$
$\kappa$	Karman constant
$\mu$	absolute viscosity of fluid, $(\text{lb})(\text{sec})/\text{ft}^2$
$\rho$	mass density, $(\text{lb})(\text{sec}^2)/\text{ft}^4$
$\tau$	shear stress in fluid, $\text{lb}/\text{ft}^2$
$\tau_t$	turbulent shear stress in fluid, $\text{lb}/\text{ft}^2$
$\tau_0$	shear stress in fluid at wall, $\text{lb}/\text{ft}^2$

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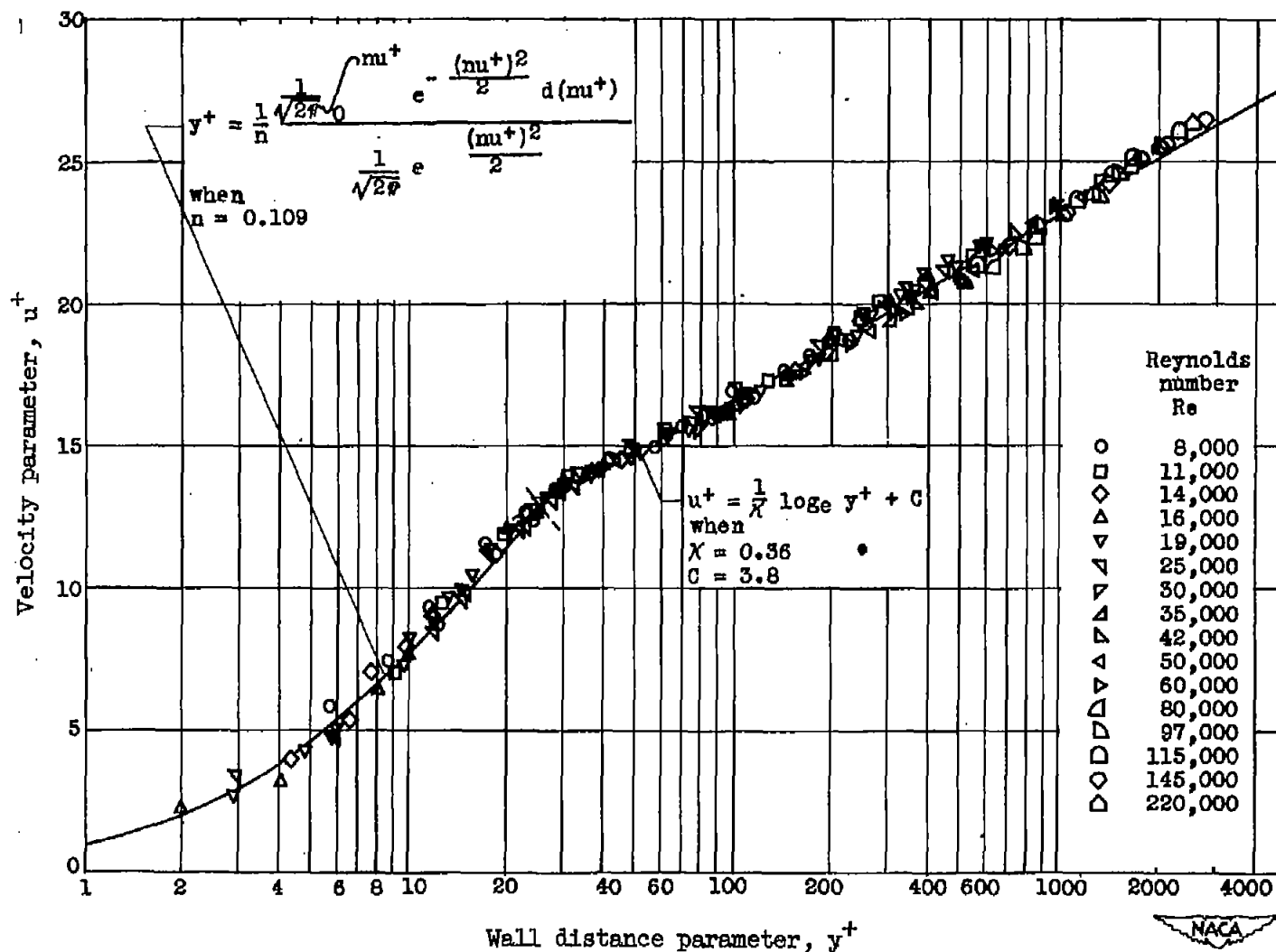


Figure 1. - Generalized velocity distribution for fully developed turbulent flow in smooth tubes (reference 10).

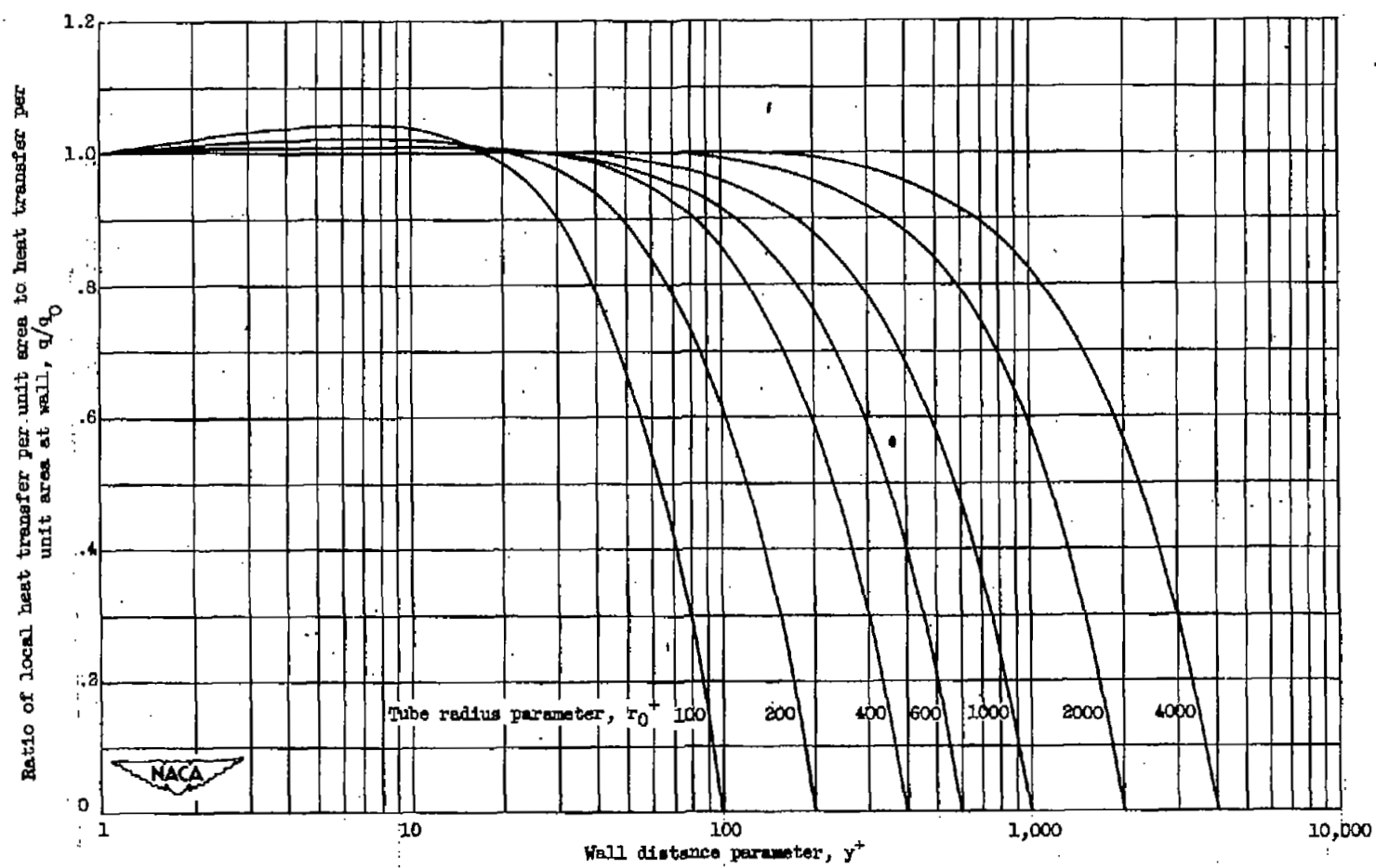


Figure 2. - Variation of heat transfer per unit area across tube.

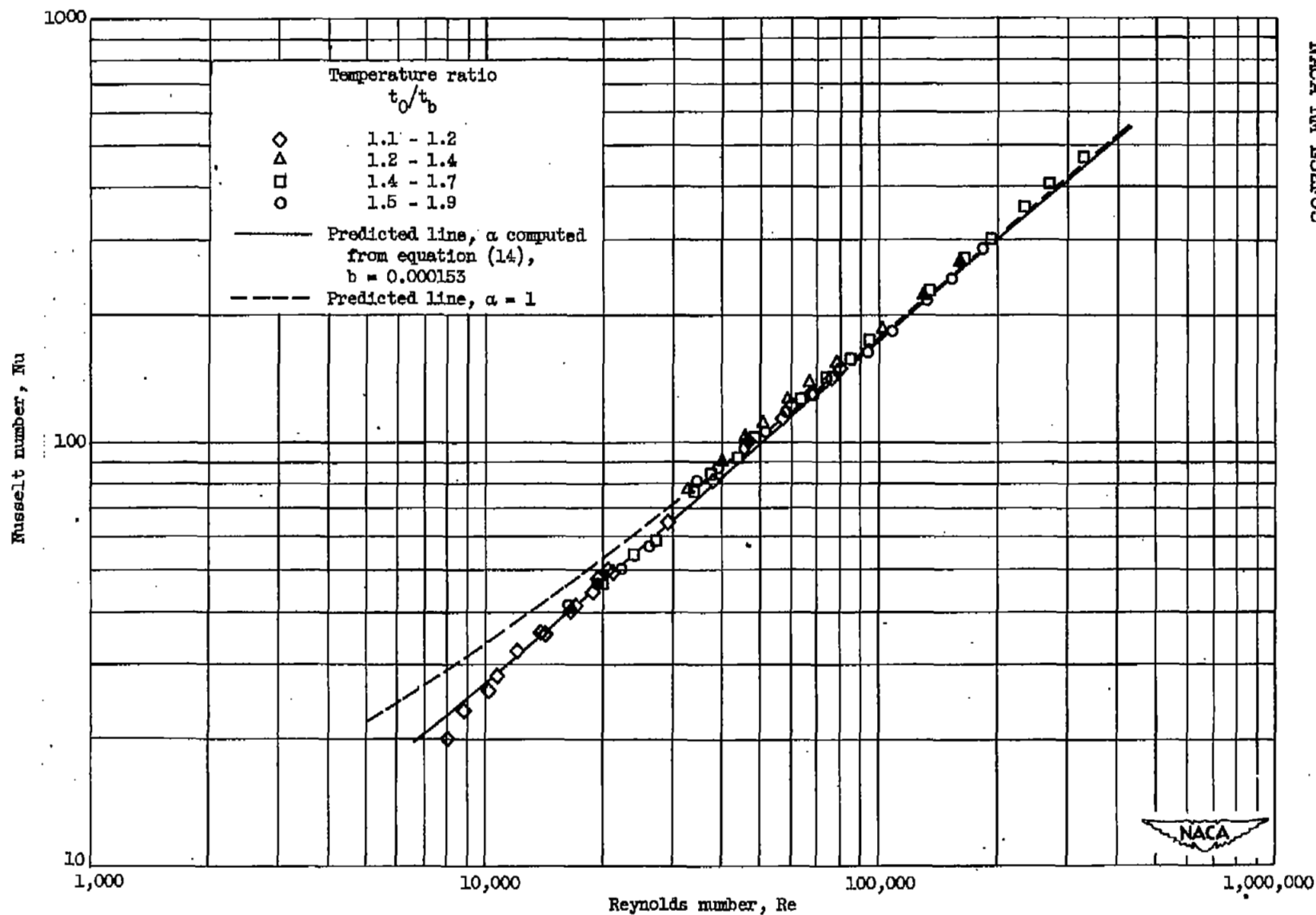


Figure 3. - Variation of Nusselt number with Reynolds number for heat transfer to air. Prandtl number, 0.73. Physical properties for data evaluated at bulk temperature for Reynolds number below 30,000 and at temperature,  $t_{0.4} = 0.4 (t_0 - t_b) + t_b$ , for Reynolds number above 30,000.

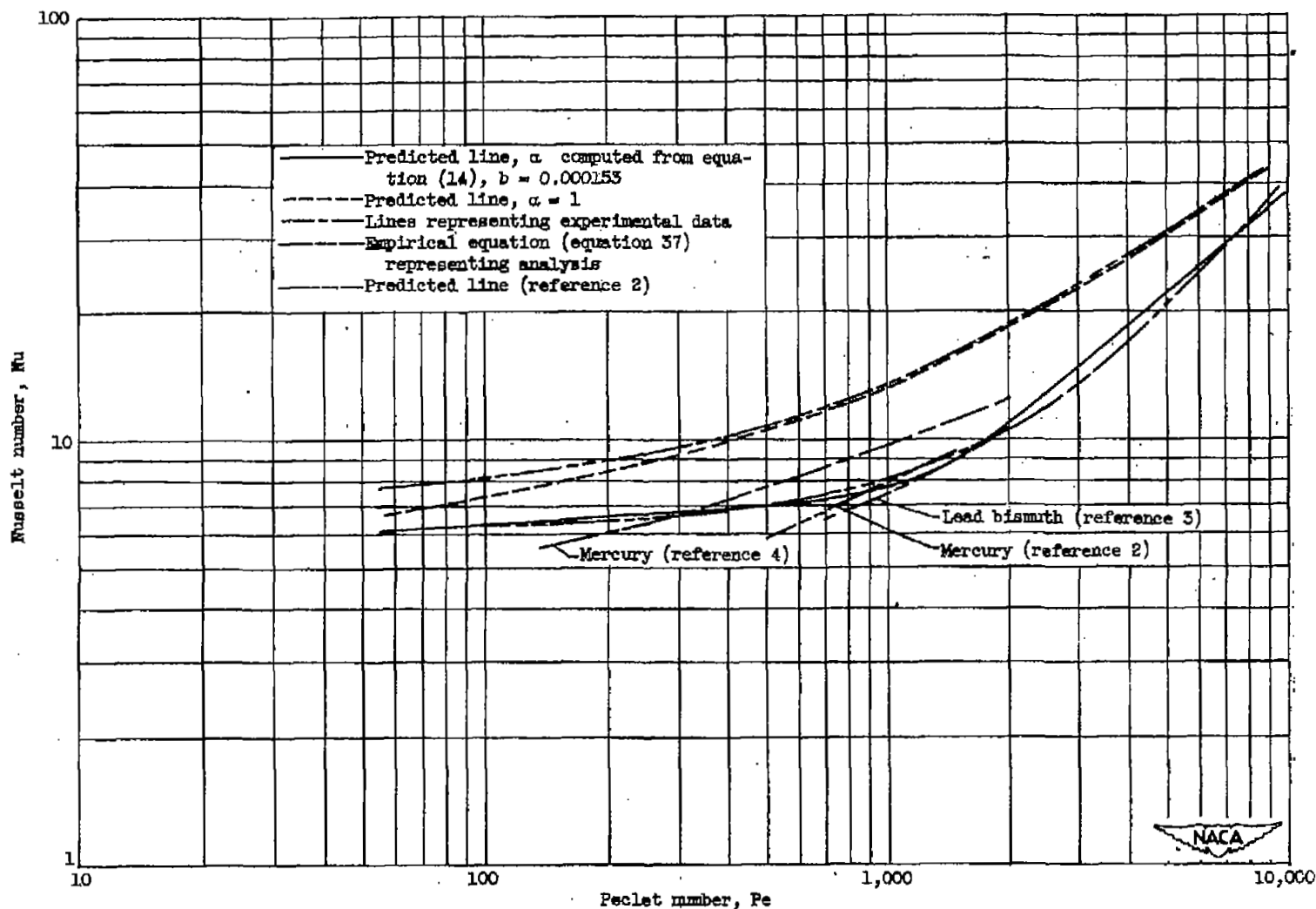


Figure 4. - Variation of Nusselt number with Peclet number for heat transfer to liquid metals at low Prandtl numbers.

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